ANALYSIS OF SPECTRAL TURBULENCE IN TWO-PHASE FLOWS

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The degeneration of the lattice turbulence in two-phase flows and the behavior of the spectrum in the space of wave numbers are analyzed on the basis of a numerical solution of the dynamic equation for the spectral function of turbulence energy.

Interest in the investigation of two-phase flows, exemplified by suspension of solids in gas and by dilute suspensions, is attributed to their wide application in metallurgy, energetics, chemical technology, and rocket engineering. In a number of practical problems it is required to know the distribution of the energy density of turbulence over wave numbers and the effect of dispersion agents on it. In the present work an investigation of the effect of particles on the energy spectrum of the carrying agent in the initial period of degeneration is conducted. The case of isotropic turbulence is considered.

It is assumed that the size of particles is less than the internal scale of turbulence λ and that the volume density of particles $\beta \ll 1$. The problem of the influence of particles suspended in a liquid on the final period of degeneration of isotropic turbulence and the distortions of the energy spectrum are considered in [1, 2], were equations of the Kármán–Howarth type for a liquid [3] and the equation for the spectral function of the turbulence energy are obtained neglecting the transport function W.

According to [1] the dynamic equation for the three-dimensional spectral function E of the energy of turbulence can be written in the form

$$\frac{\partial}{\partial t}\int_0^k E(k;\ t)\,dk = W - 2\nu_0\int_0^k k^2E(k;\ t)\,dk - \varepsilon', \tag{1}$$

where ε' is the additional dissipation of energy due to noncoincidence of fluctuating velocities of the liquid and particles.

In order to determine the transport function we make use of the Heisenberg hypothesis [3], which states that the energy transport along the spectrum of wave numbers occurs as if there existed a certain turbulence visosity $\nu_T(k;\ t)$:

$$W = -2\nu_T(k;\ t)\int_0^k k^2E(k;\ t)\,dk. \tag{2}$$

An investigation of isotropic turbulence based on the Heisenberg hypothesis is conducted in [4-6] and a detailed review is given in [3]. For the liquid ($\beta = 0$), homogeneous with respect to $\nu_T(k;\ t)$, there are different dependences in the literature constructed from intuitive considerations (see, for example, [3]).

In [7], a model of turbulent motion leading to a completely determined functional dependence of the turbulence viscosity on the spectral function is proposed, and corrections to the spectral tensor of fluctuations and the turbulence viscosity due to the presence of suspended particles are calcualted. Taking account of [7], W and ε' are of the form

$$W = -2(\nu_T + \nu^*)\int_0^k k^2E(k;\ t)\,dk, \tag{3}$$

$$\varepsilon' = 2\beta\frac{(1-\varkappa)^2}{\varkappa\left(1+\dfrac{\varkappa}{2}\right)}\rho_f\nu_T(k)\int_0^k k^2E(k;\ t)\,dk. \tag{4}$$

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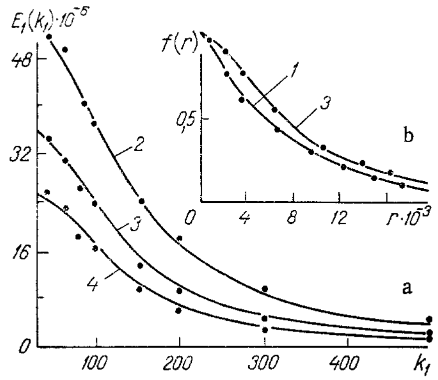

Fig. 1

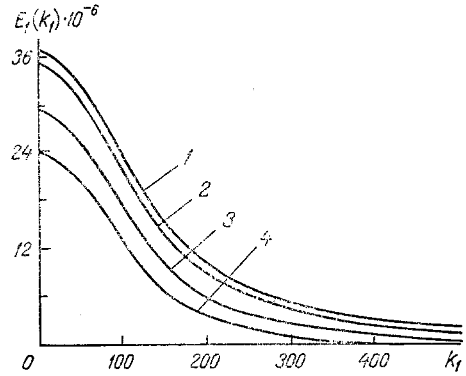

Fig. 2

Fig. 1. One-dimensional spectrum (a) and two-point correlation function (b) at different distances from the lattice (Re = 5300; curves, calculation; points, experiment): 1) x/M = 30; 2) 40; 3) 60; 4) 80.

Fig. 2. Variation of the one-dimensional spectral function with weighted concentration of particles for x/M = 60: 1) μ = 0; 2) 0.1; 3) 0.5; 4) 1.

The value ν_T is the turbulence viscosity in a pure liquid:

$$\nu_T(k;\ t) = -\nu_0 + \left[\nu_0^2 + \frac{4}{3}\alpha \int_k^\infty \frac{E(k;\ t)}{k^2} dk\right]^{1/2}, \tag{5}$$

and the correction to the turbulence viscosity due to the presence of particles is of the form

$$\nu^* \approx -\frac{4}{3}\left(\frac{1-\varkappa}{\varkappa}\right)\alpha\beta\psi^{-\beta}\int_k^\infty \psi^\beta \frac{E(k)}{\nu_T k^2} dk, \tag{6}$$

$$\psi(k) = \nu_T^2(k).$$

When the molecular viscosity is neglected, Eq. (5) changes to one of the dependences obtained by Stewart and Townsend [3] from dimensional considerations.

In order to find the value of the constant α in (5) and (6) the dynamics of degeneration of the lattice turbulence for β = 0 was calculated for experimental conditions [8].

As the initial condition E(k; t = 0) for Eq. (1) in the region $k \le k_d$ the following function was taken [6]:

$$E(k) = A\frac{(k/k_d)^4}{1+\left(B\dfrac{k}{k_d}\right)^{17/3}},\quad k_d = \frac{1}{\eta} = \left(\frac{\varepsilon}{\nu_0^3}\right)^{1/4}, \tag{7}$$

which resulted in the dependence $E \sim k^4$ in the region of small k and in the equilibrium region $E \sim k^{-5/3}$. In the region of large wave numbers, the following approximation was taken [3]:

$$E = C\varepsilon^{2/3}k_d^{-5/3}\left(\frac{k}{k_d}\right)^{-5/3}\exp\left[-D\left(\frac{k}{k_d}\right)^2\right]. \tag{8}$$

In order to determine the coefficients in (7) and (8), we used the conditions of continuity of E and ∂E/∂k for $k = k_d$ and the relations

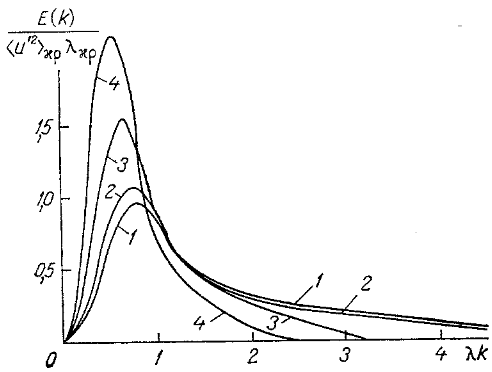

Fig. 3

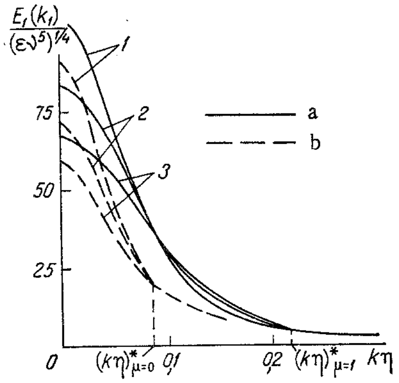

Fig. 4

Fig. 3. Normalized three-dimensional spectral functions at a point x/M = 60 for different weighted concentrations of particles: 1) $\mu = 0$; 2) 0.1; 3) 0.5; 4) 1.

Fig. 4. Distortion of the energy spectrum of the carrier phase after adding particles into the flow; 1) x/M = 40; 2) 60; 3) 80; a) $\mu = 1$; b) 0.

$$\int_0^\infty E(k;\ 0)\,dk = \frac{3}{2}\langle u'^2\rangle;\quad \varepsilon = 2\nu_0 \int_0^\infty k^2 E(k;\ 0)\,dk,$$

in which $\langle u'^2\rangle$ and ε were taken from the experimental data [8] for the distance from the lattice x/M = 30. In numerical calculations the intervals in time and in the wave space were taken in accordance with the recommendations of [6].

The one-dimensional spectral function $E_1\ (k_1;\ t)$ and the function of two-point correlation f(r) were calculated from the formulas

$$E_1(k_1;\ t) = \frac{1}{2}\int_{k_1}^\infty \frac{E(k;\ t)}{k}\left(1 + \frac{k_1^2}{k^2}\right)dk,$$

$$f(r) = 2\int_0^\infty \left[-\frac{\cos kr}{(kr)^2} + \frac{\sin kr}{(kr)^3}\right]E(k;\ t)\,dk.$$

A comparison of the calculated and experimental results [8] is shown in Fig. 1. Such agreement of the calculation and experiment was obtained for $\alpha = 0.45$, which is in good agreement with the data [5, 6].

For the case of an inhomogeneous flow with particles, a situation in which uniformly concentrated particles were injected in the flow of liquid with the developed turbulence (x/M = 30 for the conditions of the experiments [8]) was modeled, and the influence of particles on the degeneration of the lattice trubulence was elucidated.

Unfortunately, the authors did not succeed in finding experimental results on degeneration of the isotropic turbulence for two-phase flows in the literature; in connection with this, a qualitative analysis was conducted on the basis of Eqs. (1), (3)-(6).

Calculations have shown that the degeneration of the kinetic energy during the initial period follows a linear law

$$\langle u'^2\rangle = \text{const}\, t^{-(1+\mu)}, \qquad (9)$$

where $\mu = \rho/\kappa$ is a weighted concentration of particles.

Figure 2 shows one-dimensional spectra E_1 for x/M = 60, and Fig. 3 shows three-dimensional functions of the energy spectrum normalized over the corresponding values $\langle \mu'^2\rangle$ and λ for different weighted concentrations of particles. From these results it follows that the energy in the region of dissipation decreases when particles are added in the gas flow. A

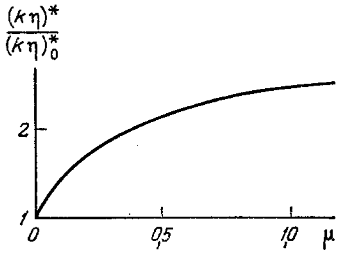

Fig. 5. Shift of the starting point of self-similar behavior of spectra as a function of the weight concentration of particles.

similar qualitative picture was observed in experiments [9, 10] on the axis of a two-phase jet.

An investigation of the variation of the Kolmogorov scale $\eta \sim 1/k_d$ for different weighted concentrations has shown that as μ increases the region of dissipation shifts toward small wave numbers and the interval $E \sim k^{-5/3}$ in the wave space decreases. A similar picture is discussed in [9]. The ratio of $\eta(\mu)$ to $\eta(\mu = 0)$ is described for small μ by an approximate dependence $\eta(\mu)/\eta(\mu = 0) \approx 1.0 + 2.15\ \mu$.

According to calculations, when considering self-similarity of one-dimensional spectra in the parameter $(\varepsilon\nu_0^5)^{1/4}$, one can observe a shift of the value $(k\eta)^*$ (the value starting from which spectra can be treated as self-similar) toward an increase of wave numbers. This is shown in Fig. 4. For the case of a homogeneous flow, the calculated one-dimensional spectra and experimental spectra [8] are self-similar not only in the region of universal equilibrium but also in the region of energy-containing eddies.

When particles are added into the flow, self-similarity is observed only in the region of viscous dissipation. The ratio of $(k\eta)^*$ in a two-phase flow to analogous value $(k\eta)_0$ for a pure liquid is shown as a function of the weight concentration of particles in Fig. 5.

In conlcusion we note that an estimate $\partial E(k;\ t)/\partial t$ and dependence (9) indicate an additional damping not only in the final period of degeneration [1, 2], but also at the initial stage with additional damping $\sim t^{-\mu}$.

NOTATION

λ, internal turbulence scale; β, volumetric concentration of particles; E, function of a three-dimensional energy spectrum; t, time; k, wave number; ν_0, kinematic coefficient of molecular viscosity; $\varkappa = \rho_f/\rho_p$; ρ_p, density of material of particles; ρ_f, liquid density; $\eta = (\nu^3/\varepsilon)^{1/4}$, Kolmogorov scale; u', pulsation velocity of the liquid; M, parameter of the turbulence-producing lattice; x, distance downward from the lattice along the flow; μ, weight concentration of particles; ε, dissipation energy.

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